A Mechanical Analysis of Simple Hook Design

Quinn Bowers

1 Introduction

This paper examines the behavior of a simple hook in the context of automated cranes. The hook end effector is suspended from a winch attached to a gantry, which can be controlled programmatically. The hook is underactuated. It is useful to guarantee certain behavior of an end effector given a control scheme. In our case the desired behavior is to orient correctly with respect to a target rod, guaranteeing that the crane can then safely pick up the target rod.

2 Problem Definition

Physical Components

Here we define the components of the system:

1. **The target rod:** This is the stationary object we are attempting to hook onto. In reality, this may be just a small part of a larger payload, positioned anywhere in the x-y plane of the *world_frame*. In this analysis, it will be approximated as an infinitely thin, infinitely long line.

2. **The coordinate system:** Our analysis will be done in the fixed *test_frame*. The y-axis of this frame is defined by the target rod. The z-axis is parallel with the z-axis of the *world_frame*. The position of the x-axis is unimportant.

3. **The gantry:** In reality, this is a chassis suspended above the ground that can move in 2 dimensions. Since our control algorithm will only move it perpendicular to the target rod, we can model it instead as a cart that moves along the x-axis. We will say the the gantry is suspended at height z_{gantry} in the *test_frame*, and that it always stays on the x-axis ($y_{gantry} = 0$).

4. **The hook:** This is modeled as an infinitely thin shape as described in figure 1. A coordinate system *hook_frame* is attached to point A. The hook's center of mass is at point E in the figure, l below point A in the hook frame. Segments AB and CD have length w. The hook is suspended from the gantry by a line of length $L = z_{gantry} - l/2$.

Success Condition

We now define the success condition of the hook system:

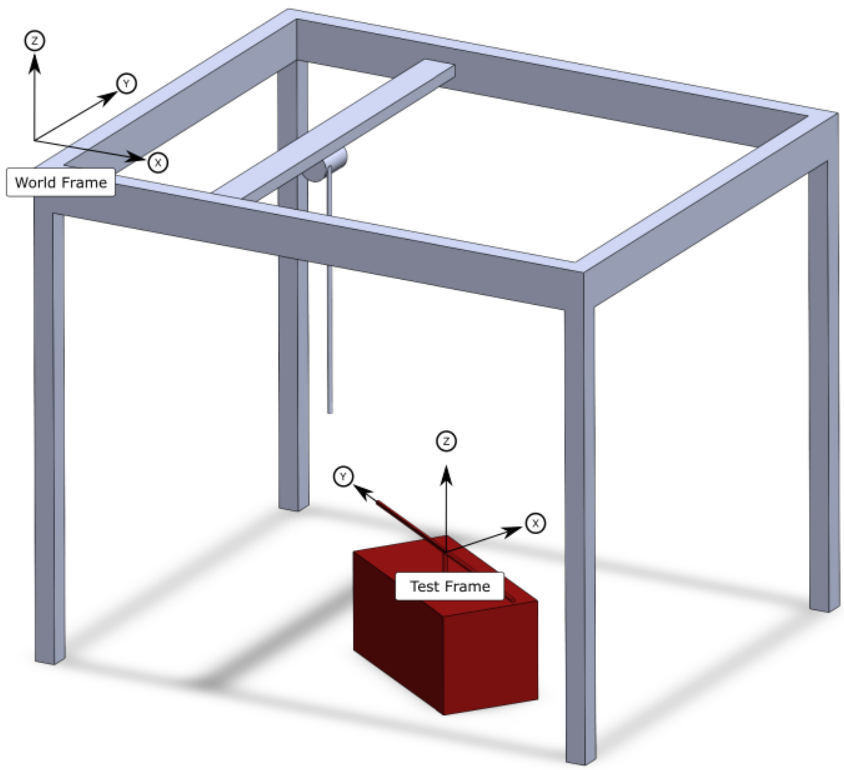

Figure 1: The grey object represents the gantry crane, fixed to the *world_frame*. The red object is an example payload with a target rod and the *test_frame* fixed to it.

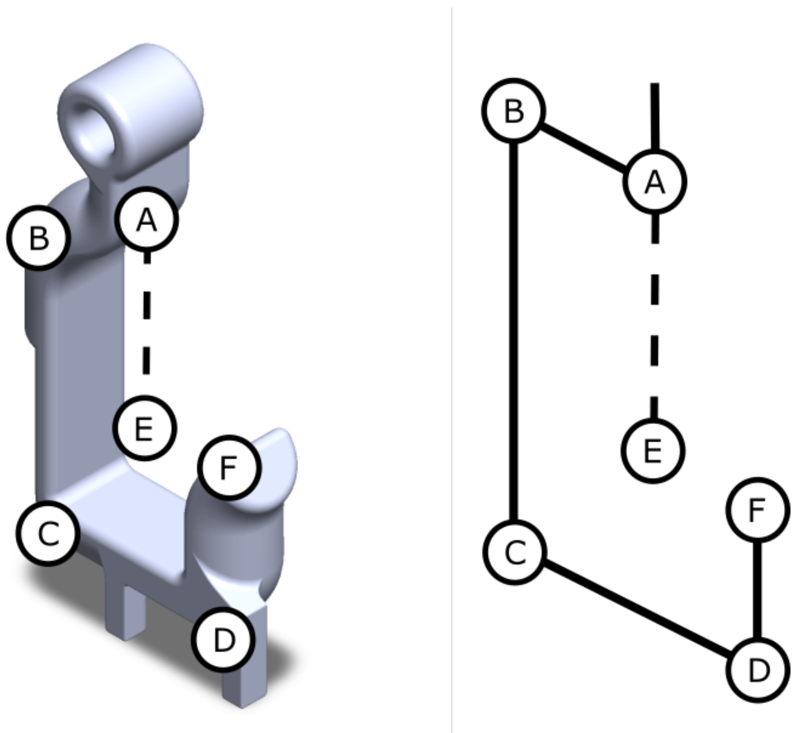

Figure 2: Left: The CAD for a physical hook used in experiments. Right: an infinitely thin approximation of the physical device.

Mechanically, we care that if the winch is retracted, the hook will not slip off the target rod. This condition occurs if the target rod is between the two vertical parts of the hook; More mathematically, if the y-axis of the *test_frame* intersects the ABCD plane. A specific case of this condition occurs if segment BC is in contact with the target rod and the rotation of the hook around the z-axis is 0. Even more specifically, this condition is true if the *hook_frame* is at position $x_{hook} = w$, $z_{hook} \in (0, l)$ with the same orientation as the *test_frame*. Lets say that the transform of the hook frame has error in each of its 6 degrees of freedom: δx, δy, δz, $\delta\theta_x$, $\delta\theta_y$, $\delta\theta_z$. The system has reached its victory condition when each of these errors reaches zero.

3 Algorithmic Overview

Initial Conditions and Assumptions

1. We assume that the system is quasi-static; that is, bodies are always in the position of lowest reachable potential energy.

2. The gantry starts in a position such that the hook cannot touch the target rod. We will say $x_{gantry} = -\infty$.

3. The *hook_frame* starts directly below the gantry:
$\vec{x}_{hook} = (x_{gantry}, 0, z_{gantry} - L)$.

4. The hook can move freely in 3D space, with the condition that it is at most L away from the gantry at all times.

The Algorithm

The gantry moves in the $\hat{x}$ direction until the center of mass is no longer directly below the gantry, or until the gantry is at position $x_{gantry} = w$. It waits until neither of these conditions is true and repeats. The program terminates immediately when the success condition is met.

Resolution of errors

Before the algorithm begins, the hook will orient such that it is in the position of lowest potential energy; specifically, the center of mass will be directly below the gantry, as low as it can go. Thus, δz, δy, $\delta\theta_x$, and $\delta\theta_y$ all become 0.

We are left with two errors to resolve: δx, and $\delta\theta_z$. The control algorithm will now resolve both at the same time.

The control algorithm now moves the system in the positive x direction. Because the system is quasi-stable, it will not pause until hook segment BC contacts the target rod; no other stimulus exists in the world, and BC is the only segment at the right height to hit the target rod. There are now three cases to consider:

a. $\delta\theta_z = 0$. This is the victory condition; the hook is in the right orientation and the program exits.

b. $\delta\theta_z = \pi$. This is the failure condition; The hook is now perfectly balanced in an inadmissable position. This condition is also unstable (see case c.) and has only infinitesimal probability of occurring. It is discounted for this analysis.

c. $\delta\theta_z \in (-\pi, pi)/0$. This is the most likely case. This case implies geometrically that $x_{gantry} < w$. When the gantry begins moving again, the contact between segment BC and the rigid target rod will impose a torque on the hook positively proportional to $w \cdot \sin \delta\theta_z$; The hook will rotate such that $\delta\theta_z$ approaches 0.

This demonstrates that if the hook is in case c., it will tend toward case a., and must necessarily reach it once $x_{gantry} = w$.

4 Potential Analysis

We can also examine this problem through the lens of potential. To explain this process, consider a 3D double pendulum: a mass hangs from a rigid rod B, which in turn hangs from another rigid rod A. If the mass has a known orientation, then this system has 5 degrees of freedom: 2 for the pitch and yaw of rod A, and 3 for the orientation of the ball joint connecting rods A and B. How would we find the equilibrium positions of such a system? For this system the answer is trivial, but we will explore more complex ways of answering the question, so that we may use these same techniques on our actual system.

One way to demonstrate that a system tends toward a point of interest $\vec{p}$ is to assume quasistatic behavior and argue that $\vec{p}$ is a global minimum of the potential function. We can assert it is an equilibrium point by creating an expression for the potential energy of the system as a function of the generalized coordinates. The gradient of the potential well must be zero evaluated at $\vec{p}$ for $\vec{p}$ to be an equilibrium point. Applying this process to the above example, we find that our expected equilibrium points are indeed also points of zero gradient in the potential well. To confirm that point $\vec{p}$ is a global minimum requires some geometric reasoning. In the case of the double pendulum, it can be seen that the global minimum is the point at which the mass is lowest in space. To confirm that the system tends toward point $\vec{p}$, we also need some understanding of the surface of potential well. This is more difficult to arrive at. We will hand-wave it away for the purposes of this analysis.

Consider now our actual system (we will examine the case were $x = w$). We predict there to be only one equilibrium point that satisfies the contact constraint, namely the position in which the hook is oriented straight down and straight forward. We can also compute the potential for this system as a function of four generalized coordinates; like the double pendulum, this system would have five degrees of freedom, but one is removed by the contact constraint. This lets us solve for one of these coordinates analytically. We can now find the gradient of our potential curve evaluated at the point of interest $\vec{p}$. This process was done in MATLAB, and intermediate steps show some pretty gross analytical expressions which we will omit for brevity. In the end, evaluating the gradient at our point of interest yeilded zero. We are confident that point $\vec{p}$ is an equilibrium point.

5 Discussion

Initial Alignment

To do this hooking algorithm practically requires identifying the location of the target rod (or the payload it is attached to). This can be done in a number of ways, but in our cases has been achieved with computer vision. A camera is mounted to the gantry crane, and detects an April Tag mounted rigidly to the target rod.

Failure Cases

The algorithm will fail if case b. does actually occur. Because physical objects will not be infinitely thin or infinitely round, there is a non-zero, if low, probability that case b. occurs. This is, however, detectable, as when the crane attempts to pick up the payload there will be no load on the cable and thus less current through the motor. The current can be measured to determine if hook was successful, and in case of failure, the system can reset and try again.

The algorithm may also fail if any of the assumptions it is built on are incorrect. The most likely to be problematic is the assumption that the hook can move freely in 3D space (except for the constraint that it is at most L away from the gantry). Due to complex cable dynamics, it is possible that it does not rotate freely around the cable.

It is also possible that the dynamics of the system are too fast to assume quasi-stability. This can be resolved by slowing the algorithm entirely.